Large-scale and infinite dimensional dynamical model approximation

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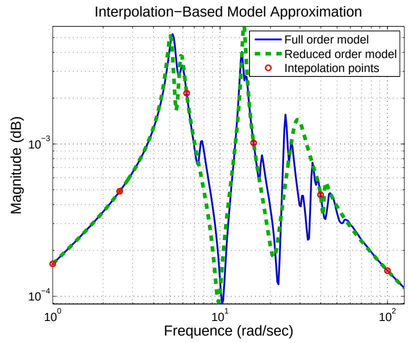
Context : This thesis addresses the $\mathcal{H}_2$ **model reduction techniques** using **interpolation-based approaches** and its application to **infinite-dimensional models**, especially to **time-delay systems**. It is a continuation Pierre Vuillemin's thesis, which was on the subject of frequency-limited model reduction, also carried out in the french aerospace lab (ONERA) at the systems control and flight dynamics department (DSCD). During this thesis, a collaboration with **S. Gugercin** and **C. Beattie** at VirginiaTech was set up, during the 3 months I have spent visiting their lab in Blacksburg (VA, United States).



Problem setting and methodology : Given a linear time-invariant (LTI) dynamical system represented by the transfer matrix $\mathbf{H} \in \mathcal{H}_2^{n_y \times n_u}$ with n_u inputs and n_y outputs, we consider the problem of finding a reduced order model $\hat{\mathbf{H}}$ which that minimizes the $\mathcal{H}_2$-error ; *i.e.*,

$$\|\mathbf{H} - \hat{\mathbf{H}}\|_{\mathcal{H}_2} = \min_{\text{order}(\mathbf{G})=r, \mathbf{G} \in \mathcal{H}_2} \|\mathbf{H} - \mathbf{G}\|_{\mathcal{H}_2}, \tag{1}$$

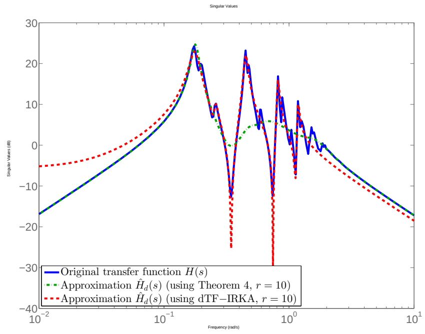
where, in general, $\hat{\mathbf{H}}$ is considered to have a finite dimensional state space structure. One natural way to construct a model approximation in the sense of $\mathcal{H}_2$ mismatch minimization is to use rational interpolation. So, given a high dimensional system $\mathbf{H}$ of order $n \gg 1$ and some interpolation points $\{\sigma_1, \ldots, \sigma_r\} \subset \mathbb{C}$, one is interested in finding a reduced order model $\hat{\mathbf{H}}$ such that (**SISO** conditions) $\mathbf{H}(\sigma_k) = \hat{\mathbf{H}}(\sigma_k)$ and $\mathbf{H}'(\sigma_k) = \hat{\mathbf{H}}'(\sigma_k)$, for $k = 1, \ldots r$. The above figure illustrates this idea, and those methods are the so-called **interpolation-based**. Indeed, the first order optimality conditions of (1) can be viewed as interpolation conditions. Over the past two decades, major progress has been made in **interpolation-based model reduction approaches** and, as a result, these methods have emerged as one of the leading choices for reducing large-scale dynamical systems. These methods fall into two categories, **projection methods** and **data-driven/Loewner methods**. The last one is based on the original model transfer function evaluations, allowing model reduction from data and realisation-less models. This framework is very useful in the case where the original model is represented by a irrational transfer function, for example, time-delay systems and system governed by PDE, also denoted as **infinite dimensional systems**. Iterative procedures using interpolation-based approaches can be set to produce (locally) optimal reduced models at modest cost, *e.g.*, **IRKA**-like algorithms.

Overview of my contribution : In this thesis, we were particularly interested on the model reduction of infinity dimensional systems, *e.g.*, time-delay systems (TDS) and irrational transfer functions. Since for this class of systems, classical analysis and control methods are not applicable, it is very appealing to approximate them to a finite order system. Even if many dedicated approaches have been derived to handle infinite dimensional problems, most of then are limited to systems with low order state vector and associated methods are not scalable when the order of the model is increasing. Many examples of them can be found in the context of network systems, where delays naturally appear as the amount of time necessary to transmit some information between different systems (communication lag). In other systems, they are intrinsic part of the natural phenomena as it can be seen in chemical reactions, traffic jam and heating systems...

The aim of this thesis is to investigate the approximation of large-scale systems and infinite dimension systems. We are particularly interested in the problem using **data-driven/Loewner** interpolation methods. My work has been focused on new applications of those model approximation techniques for control, analysis and developing new model reduction techniques for reduced order models with richer structures. The originally of this work comes from the combination of results from two different fields : Time-delay systems and model approximation. Although it is a model approximation thesis, we believe that many results developed here can be useful in other fields, *e.g.*, the spectral $\mathcal{H}_2$ inner product computation for input/output and state-delay models [3] and [2]. The following list summarize my main contributions and publications. It is organized in **Theoretical Contributions**, **Methodological contributions** and **Model reduction and applications**.

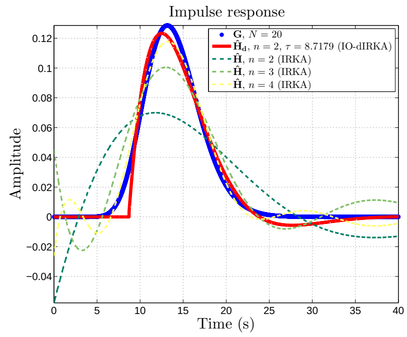
1)Theoretical Contributions :

a) Model reduction for state-delay : We have developed a generalization of the Loewner framework, initially settled for delay-free realization, to the single state-delay case, *i.e.*, when the reduced order model has the transfer function with the following delay structure $\hat{\mathbf{H}}_{\mathbf{d}} = \hat{\mathbf{C}}(\hat{\mathbf{E}}s - \hat{\mathbf{A}}e^{-s\tau})^{-1}\hat{\mathbf{B}}$.



Secondly, using the Lambert function we develop a new algorithm called **dTF-IRKA** enabling to construct a reduced order delay-model satisfying some interpolation conditions. Theoretical results and several numerical examples are presented in the CDC conference paper [4].

✓ delay-Lowner framework ✓ Iterative algorithm **dTF-IRKA**



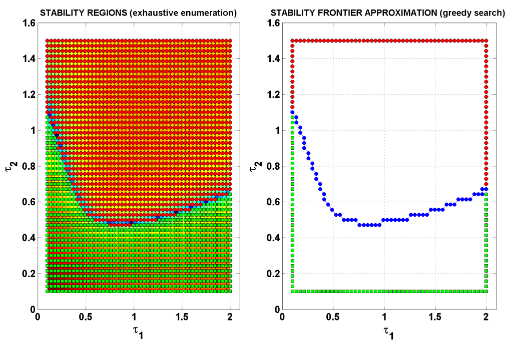
b) Model reduction for input/output-delay structure : Given $\mathbf{G}$ a finite dimensional stable model , we have studied the $\mathcal{H}_2$ optimal model approximation problem where the reduced order model include input/output delays, *i.e.*, $\hat{\mathbf{H}}_\mathbf{d} = \Delta_o(s)\hat{\mathbf{C}}(\hat{\mathbf{E}}s - \hat{\mathbf{A}})^{-1}\hat{\mathbf{B}}\Delta_i(s)$. The delta blocks here represent multiple input and output delays having the form $\Delta_{i/o}(s) = \mathbf{diag}(e^{-\tau_1 s}, \ldots, e^{-\tau_M s})$. We found the $\mathcal{H}_2$ optimality conditions of this problem based on the poles/residues decomposition. Finally, a two stage algorithm in order to practically obtain such an approximation has been proposed. Those results were submitted to the Automatica journal [3]. Moreover, a paper setting the parallel between those results and Lyapunov equations is also planned.

✓ $\mathcal{H}_2$-optimality conditions for input/output delay structure ✓Iterative algorithm based on **IRKA**

c) More general $\mathcal{H}_2$-optimality conditions : The $\mathcal{H}_2$-optimal model reduction problem was revisited when the reduced system has a more general pole-residue structure and new $\mathcal{H}_2$-optimality conditions where obtained. These conditions are no longer simple interpolation conditions as in the finite-dimensional case and they depend on the spectral structure of the reduced model system. Preliminary results were submitted to the 2016 IFAC time-delay workshop [2] and a more general version to a journal.

✓ more general $\mathcal{H}_2$-optimality conditions

2) Methodological contributions :

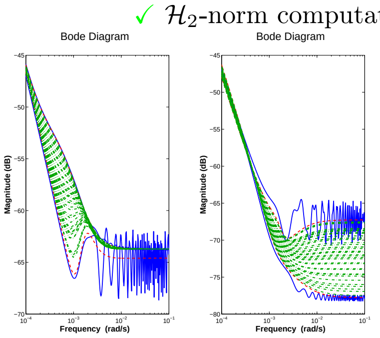


a) Model reduction and stability charts for time-delay systems : The problem of determining the approximate stability regions of large-scale time-delay systems is attacked using model approximation techniques. To achieve this, an $\mathcal{H}_2$-oriented approximation algorithm. We show how model reduction can be used to approximate time-delay systems with multiple delays and estimate their stability regions. Theoretical results and several numerical examples are presented in the ECC conference paper [7] in collaboration with C. Briat (ETH Zurich). The above figures illustrate those results. In addition, This methodology was coupled with a boundary search algorithm. This as well as some theoretical results should be submit to a journal.

✓ Stability charts using $\mathcal{H}_2$-model reduction ✓ faster - convergence speed not known

b) Model reduction and norm computation : The computation of $\mathcal{H}_2$ norms for TDS are important challenging problems for which several solutions have been provided in the literature. We propose to compute this norm using $\mathcal{H}_2$ model reduction for TDS, based on the algorithm **TF-IRKA** [5].

✓ $\mathcal{H}_2$-norm computation using model reduction - distance to unstability not known



3) Realisation-less model reduction and applications : The realisation-less interpolation based model reduction has been successfully used to some industrial application for control and analysis purpose. The effect of output delays was studied in a flexible aircraft model (Dassault's model) and the obtained results were presented in the EuroGNC conference [6].

A suitable parametric rational approximation of a fluid motion modelised by the Saint-Venant partial differential equations representing an open channel flow used for hydroelectricity (EDF's model) were submited to 2016 ECC conference [1].

✓ Application to industrial benchmarks (Dassault, EDF)

From a practical point of view, all the proposed methods and tools have been integrated in the **MORE Toolbox** [8] and have successfully been used in several industrial applications for control and analysis.

Conclusion and perspectives : During my thesis, new theoretical results and algorithms were developed in order to enrich the nature of reduced order models and the applications of model reduction. The originality of this work was to combine and develop new results from the fields of model approximation and time-delay systems. To conclude, I personally believe that model reduction is a booming field with very interesting and useful results which are still not being used in industry and applied research.

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